Instructions: Try these problems from the textbook as practice (answers below).

Problems

5-22. Find the local maximum and minimum values and the saddle point(s) of the function.

5. $f(x,y) = x^2 + xy + y^2 + y$
7. $f(x,y) = 2x^2 - 8xy + y^4 - 4y^3$
9. $f(x,y) = (x-y)(1-xy)$
11. $f(x,y) = y\sqrt{x} - y^2 - 2x + 7y$
13. $f(x,y) = x^3 - 3x + 3xy^2$
15. $f(x,y) = x^4 - 2x^2 + y^3 - 3y$
17. $f(x,y) = xy - x^2y - xy^2$
19. $f(x,y) = e^x \cos y$
21. $f(x,y) = y^2 - 2y\cos x,\ -1 \le x \le 7$

23. Show that $f(x,y) = x^2 + 4y^2 - 4xy + 2$ has infinitely many critical points and that $D = 0$ at each one. Then show that f has a local (and global) minimum at each critical point.

33-40. Find the global maximum and minimum values of f on the set D.

33. $f(x,y) = x^2 + y^2 - 2x$, D is the triangular region with vertices $(2,0)$, $(0,2)$ and $(0,-2)$.
35. $f(x,y) = x^2 + y^2 + x^2y + 4$, $D = \{(x,y) : |x| \le 1 \text{ and } |y| \le 1\}$
37. $f(x,y) = x^2 + 2y^2 - 2x - 4y + 1$, $D = \{(x,y) : 0 \le x \le 2, 0 \le y \le 5\}$
39. $f(x,y) = 2x^3 + y^4$, $D = \left\{(x,y) : x^2 + y^2 \le 1\right\}$

43. Find the shortest from the point $(2,0,-3)$ to the plane $x + y + z = 1$.

45. Find the points on the cone $z^2 = x^2 + y^2$ that are closest to the point $(4,2,0)$.

47. Find three positive numbers whose sum is 100 and whose product is a maximum.

49. Find the maximum volume of a rectangular box that is inscribed in a sphere of radius r.

51. Find the volume of the largest rectangular box in the first octant with three faces in the coordinate planes and one vertex in the plane $x + 2y + 3z = 6$.

53. Find the dimensions of a rectangular box of maximum volume such that the sum of the lengths of its 12 edges is a constant c.

55. A cardboard box without a lid is to have a volume of $32,000\text{cm}^3$. Find the dimensions that minimize the amount of cardboard used.

Answers

5. Minimum $f(1/3, -2/3) = -1/3$.
7. Minima $f(-2, -1) = -3$ and $f(8, 4) = -128$. Saddle at $(0, 0)$.
9. Saddles at $(1, 1)$ and $(-1, -1)$.
11. Maximum $f(1, 4) = 14$.
13. Maximum $f(-1, 0) = 2$. Minimum $f(1, 0) = -2$. Saddles at $(0, 1)$ and $(0, -1)$.
15. Maximum $f(0, -1) = 2$. Minima $f(-1, 1) = f(1, 1) = -3$. Saddles at $(0, 1)$, $(1, -1)$, and $(-1, -1)$.
17. Maximum $f(1/3, 1/3) = 1/27$. Saddles at $(0, 0)$, $(1, 0)$, and $(0, 1)$.
19. None.
21. Minima $f(0, 1) = f(\pi, -1), f(2\pi, 1) = -1$. Saddles at $(\pi/2, 0)$ and $(3\pi/2, 0)$.
33. Maxima $f(0, -2) = f(0, 2) = 4$. Minimum $f(1, 0) = -1$.
35. Maxima $f(-1, 1) = f(1, 1) = 7$. Minimum $f(0, 0) = 4$.
37. Maxima $f(0, 3) = f(2, 3) = 7$. Minimum $f(1, 1) = -2$.
39. Maximum $f(1, 0) = 2$. Minimum $f(-1, 0) = -2$.
43. $2/\sqrt{3}$
45. $(2, 1, \sqrt{5})$ and $(2, 1, -\sqrt{5})$
47. $x = y = z = 100/3$
49. $\frac{8}{9}\sqrt{3}r^3$
51. $4/3$
53. $x = y = z = c/12$
55. Length, width $x = y = 40$cm, and height $z = 20$cm.